Faraday's Law of Electromagnetic Induction

Faraday's Law :- It states that " the magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through it & its direction opposes the flux change ".

$$\boxed{e = -\frac{d\phi}{dt}} \quad \text{--- (1)}$$

Emf also indicates a voltage about a closed path such that if any part of the path is changed emf will also change.

$$\boxed{e = \oint \vec{E} \cdot \vec{dl}} \quad \text{--- (2)}$$

from (1) & (2)

$$\boxed{\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi}{dt}}$$

i) closed circuit is stationary & magnetic field is varying with time. which emf induced is

where $\phi = \int_s \vec{B} \cdot \vec{ds}$.

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt}\left[\int_s \vec{B} \cdot \vec{ds}\right]$$

$$\boxed{\oint \vec{E} \cdot \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}} \quad \text{--- (3)}.$$

Eq (3) is faraday's Law in integral form.

from stoke's theorem.

$$\oint \vec{E} \cdot \vec{dl} = \int_s (\nabla \times \vec{E}) \vec{ds} \quad \text{--- (4)}.$$

bring eq (4) in (3).

$$\int_s (\nabla \times \vec{E}) \cdot \vec{ds} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

$$\Rightarrow \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \text{--- (5)}$$

This is point form of faraday's law.

ii) magnetic field is stationary while closed circuit is revolving.

Since $\vec{B}$ is constant.

equation (3) & (5) reduces to.

$$\oint \vec{E} \cdot \vec{dl} = 0.$$

$$\nabla \times \vec{E} = 0.$$

Consider a charge Q is moved in a magnetic field $\vec{B}$ at a velocity $\vec{V}$ then.

$$\vec{F} = Q\vec{V} \times \vec{B}$$

$$\vec{E}_m = \frac{\vec{F}}{Q} = \vec{V} \times \vec{B}$$

thus emf is given by

$$\boxed{\oint \vec{E} \cdot \vec{dl} = \oint (\vec{V} \times \vec{B}) \cdot \vec{dl}} \quad \text{--- (6)}$$

iii) If magnetic flux density is varying & closed circuit is also revolving then induced emf is sum of eq (3) & (6)

$$\boxed{\oint \vec{E} \cdot \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} + \oint (\vec{V} \times \vec{B}) \cdot \vec{dl}}$$

Concept of Displacement Current.

from ampere circuital law.

$$\nabla \times \vec{H} = \vec{J}. \quad \text{--- (1)}$$

taking divergence on both sides.

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}.$$

W.K.T $\nabla\cdot(\nabla\times\vec{H}) = 0.$

$$\Rightarrow \nabla\cdot\vec{J} = 0. \quad \text{--- (2)}$$

but from continuity equation.

$$\nabla\cdot\vec{J} = -\frac{\partial\rho_v}{\partial t}. \quad \text{--- (3)}$$

eq (2) is true only when

$$\frac{\partial\rho_v}{\partial t} = 0 ..$$

thus equations (3) & (2) are not compatible for time varying fields.

∴ eq(1) must be modified by adding an unknown term 'N'.

∴ eq(1) becomes,

$$\nabla\times\vec{H} = \vec{J} + \vec{N}.$$

taking divergence on both sides.

$$\nabla\cdot(\nabla\times\vec{H}) = \nabla\cdot\vec{J} + \nabla\cdot\vec{N}.$$

$$0 = \nabla\cdot\vec{J} + \nabla\cdot\vec{N}.$$

$$\nabla\cdot\vec{N} = -\nabla\cdot\vec{J}$$

$$\nabla\cdot\vec{N} = -\left(-\frac{\partial\rho_v}{\partial t}\right)$$

$$\nabla\cdot\vec{N} = \frac{\partial\rho_v}{\partial t}. \quad \text{--- (4)}$$

from gauss law, $\rho_v = \nabla\cdot\vec{D}$ --- (5)

using (5) in (4)

$$\nabla\cdot\vec{N} = \frac{\partial}{\partial t}(\nabla\cdot\vec{D}).$$

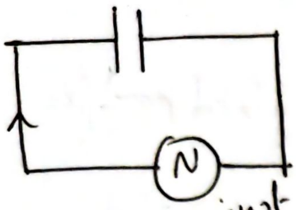
$$\nabla\cdot\vec{N} = \nabla\cdot\frac{\partial\vec{D}}{\partial t}.$$

$$\boxed{\vec{N} = \frac{\partial\vec{D}}{\partial t}}$$

$$\therefore \boxed{\nabla \times \vec{H} = \vec{J}_C + \vec{J}_D}$$

J_D = displacement current density.

* Equivalence of conduction current & displacement current



$V_0 e^{j\omega t}$.

Consider a parallel plate capacitor connected across an a.c source. Let 'A' be the area of plates & 'd' be the distance of separation.

i) Conduction current

For parallel plate capacitor

$$I = C \frac{dv}{dt}. \quad \text{where} \quad C = \frac{\epsilon A}{d} \; \& \; V = V_0 e^{j\omega t}.$$

$$I = \frac{\epsilon A}{d} \frac{d}{dt} \left(V_0 e^{j\omega t} \right)$$

$$= \frac{\epsilon A}{d} V_0 \, j\omega \, e^{j\omega t}.$$

$$\boxed{I = \frac{j\omega \epsilon A}{d} V_0 e^{j\omega t}} \quad -(1)$$

ii) displacement current.

the displacement current density is $\frac{\partial D}{\partial t}$.

we have $D = \epsilon E$.

for parallel plate capacitor $E = V/d$.

$$\therefore D = \frac{\epsilon V}{d},$$

if I_D is the displacement current, then $J_D = \frac{I_D}{A}$.

$$\Rightarrow \frac{I_D}{A} = \frac{\partial D}{\partial t}.$$

$$\Rightarrow I_D = A \cdot \frac{\partial D}{\partial t}.$$

$$= A \cdot \frac{\partial}{\partial t}(\epsilon E) = A\epsilon \frac{\partial}{\partial t}\left(\frac{V}{d}\right).$$

$$= \frac{A\epsilon}{d} \frac{\partial}{\partial t}\left(V_0 e^{j\omega t}\right).$$

$$= \frac{\epsilon A}{d} V_0 \, j\omega \, e^{j\omega t}.$$

$$\boxed{I_D = \frac{j\omega \epsilon A}{d} V_0 e^{j\omega t}} \quad -(2).$$

from (1) & (2).

Conduction current = displacement current.

Ratio of magnitudes of conduction current density to displacement current density

we have from ohm's law

$$J = \sigma E$$

For a time varying field $E = E_0 e^{j\omega t}$.

$$\therefore J = \sigma E_0 e^{j\omega t} \quad -(1)$$

The displacement current density is

$$\frac{\partial D}{\partial t} = \frac{\partial(\epsilon E)}{\partial t} = \epsilon \frac{\partial E}{\partial t} = \epsilon \frac{\partial (E_0 e^{j\omega t})}{\partial t} = \epsilon E_0 j\omega e^{j\omega t} - (2)$$

taking the ratio of 1 & 2.

$$\frac{J}{(\partial D/\partial t)} = \frac{\sigma E_0 e^{j\omega t}}{\epsilon E_0 j\omega e^{j\omega t}} = \frac{\sigma}{j\omega\epsilon}.$$

$\therefore$ the magnitude of the ratio is.

$$\left| \frac{J}{(\partial D/\partial t)} \right| = \frac{\sigma}{\omega\epsilon}.$$

(1) A circular loop conductor lies in plane $z=0$ & has a radius of 0.1 m & resistance of 5Ω. Given $\vec{B} = 0.2 \sin 10^3 t \, \hat{a}_z$ T. Determine the current in the loop. (6)

:— Current $i = \dfrac{\text{induced emf}}{\text{resistance}}$.

$$e = -\frac{d\phi}{dt}$$

we know, $\phi = \int_s \vec{B} \cdot \vec{ds}$.

$$\phi = \int_{r=0}^{0.1} \int_{\phi=0}^{2\pi} (0.2 \sin 10^3 t)\, \hat{a}_z \cdot (r\,dr\,d\phi)\, \hat{a}_z \,.$$

$$\phi = 0.2 \sin 10^3 t \int_{\phi=0}^{2\pi} d\phi \int_{r=0}^{0.1} r\,dr \,.$$

$$\phi = 0.2 \sin 10^3 t \; [\phi]_0^{2\pi} \left[\frac{r^2}{2}\right]_0^{0.1}$$

$$\phi = 0.2 \sin 10^3 t \cdot [2\pi - 0] \left[\frac{0.1^2}{2} - 0\right]$$

$$\phi = 0.2 \sin 10^3 t \; (2\pi) \left(\frac{0.1^2}{2}\right).$$

$$\phi = 6.283 \times 10^{-3} \sin 10^3 t \,.$$

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[6.283 \times 10^{-3} \sin 10^3 t\right].$$

$$\boxed{e = -6.283 \times 10^{-3+3} \cos 10^3 t} \quad V.$$

$$i = \frac{-6.283 \times \cancel{10^3} \cos 10^3 t}{5}.$$

$$\boxed{i = -1.2567 \cos 10^3 t} \quad A.$$

* Maxwell's Equations

Maxwell's equations are nothing but a set of four expressions derived from ampere's circuital law, faraday's law, gauss law for electric field & gauss law for magnetic fields.

These four expressions can be written in the following forms.

i) point or differential form.

ii) integral form.

→ From Ampere's circuital law.

$$\oint \vec{H} \cdot \vec{dl} = I_{enclosed}$$

$$\oint \vec{H} \cdot \vec{dl} = \int_S \vec{J} \cdot \vec{ds}$$

Above expression can be made further general by adding displacement current density to conduction current density as follows.

$$\boxed{\oint \vec{H} \cdot \vec{dl} = \int_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t}\right] \cdot \vec{ds}} \quad — (1)$$

eq (1) is maxwell's eqn derived from ampere's circuital law.

Applying stoke's theorem to eq (1).

$$\int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot \vec{ds}$$

$$\Rightarrow \boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}} \quad — (2)$$

This is point form of maxwell's eqn derived from ampere's circuital law.

→ From faraday's law.

$$\boxed{\oint \vec{E}\cdot\vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}} \quad \text{— (3)}$$

eq(3) is integral form of maxwell's eqn derived from faraday's law.

Converting line integral to surface integral in eq (3)

$$\int_s (\nabla\times\vec{E})\cdot\vec{ds} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}.$$

$$\Rightarrow \boxed{\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \text{— (4)}$$

This is point form of maxwell's eqn derived from faraday's law.

→ From gauss law for electric field,

$$\int_s \vec{D}\cdot\vec{ds} = Q_{enclosed}$$

Expressing Q in terms of volume integral

$$\boxed{\int_s \vec{D}\cdot\vec{ds} = \int_v \rho_v \, dv.} \quad \text{— (5)}$$

eq (5) is integral form of maxwell's eqn for electric field derived from gauss law.

using divergence theorem.

$$\int_v (\nabla\cdot\vec{D})\,dv = \int_v \rho_v \, dv.$$

$$\Rightarrow \boxed{\nabla\cdot\vec{D} = \rho_v.} \quad \text{— (6)}$$

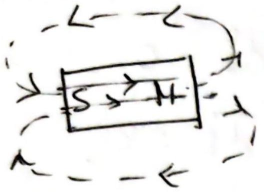
eq (6) is point form of maxwell's equation for electric field derived from gauss law.

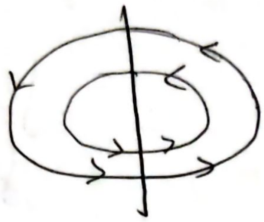
* Gauss Law for magnetic field

For closed surface, magnetic flux is zero.

i.e $$\boxed{\oint_s \vec{B}\cdot d\vec{s} = 0}$$ — (1)

because magnetic field always stays in closed loop.





If I consider a surface flux entering is equal to flux leaving ∴ total flux is equal to zero.

Equation (1) is integral form of maxwell's equation for magnetic fields derived from Gauss law.

using divergence theorem to equation (1).

$$\oint_s \vec{B}\cdot d\vec{s} = \int_v (\nabla\cdot\vec{B})\,dv = 0$$

$$\Rightarrow \boxed{\nabla\cdot\vec{B} = 0} \quad — (2)$$

Equation (2) is point form of maxwell's equation for magnetic field derived from Gauss law.

Maxwell's Equations

1. For time varying fields:

Differential form	integral form
i) $\nabla\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$	$\oint \vec{E}\cdot d\vec{l} = -\int_s \dfrac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$
ii) $\nabla\times\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$	$\oint \vec{H}\cdot d\vec{l} = \int_s \left[\vec{J} + \dfrac{\partial \vec{D}}{\partial t}\right] d\vec{s}$
iii) $\nabla\cdot\vec{D} = \rho_v$	$\oint \vec{D}\cdot d\vec{s} = \int \rho_v\, dv$
iv) $\nabla\cdot\vec{B} = 0$	$\oint \vec{B}\cdot d\vec{s} = 0$

2. For free space $\Rightarrow$ $\rho_v = 0$, $\vec{J} = 0$.

Differential / point	Integral form
i) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.	$\oint \vec{E} \cdot \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$.
ii) $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$	$\oint \vec{H} \cdot \vec{dl} = \int_s \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$.
iii) $\nabla \cdot \vec{D} = 0$	$\oint_s \vec{D} \cdot \vec{ds} = 0$.
iv) $\nabla \cdot \vec{B} = 0$	$\oint_s \vec{B} \cdot \vec{ds} = 0$.

3. For good conductor $\Rightarrow$ $\rho_v = 0$, $\vec{J} \gg \frac{\partial \vec{D}}{\partial t}$.

Differential form	integral form.
i) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.	$\oint \vec{E} \cdot \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$.
ii) $\nabla \times \vec{H} = \vec{J}$	$\oint \vec{H} \cdot \vec{dl} = \int_s \vec{J} \cdot \vec{ds}$.
iii) $\nabla \cdot \vec{D} = 0$	$\oint_s \vec{D} \cdot \vec{ds} = 0$.
iv) $\nabla \cdot \vec{B} = 0$	$\oint_s \vec{B} \cdot \vec{ds} = 0$.

4. For harmonically varying field

$\vec{D} = \vec{D_0} \cdot e^{j\omega t}$. $\quad \vec{B} = \vec{B_0} \cdot e^{j\omega t}$.

Differential form	integral form.
i) $\nabla \times \vec{E} = -j\omega\mu \vec{H}$	i) $\oint \vec{E} \cdot \vec{dl} = -j\omega\mu \int_s \vec{H} \cdot \vec{ds}$.
ii) $\nabla \times \vec{H} = (\sigma + j\omega\epsilon) \vec{E}$	ii) $\oint \vec{H} \cdot \vec{dl} = \int_s (\sigma + j\omega\epsilon) \vec{E} \cdot \vec{ds}$.
iii) $\nabla \cdot \vec{D} = \rho_v$	iii) $\oint_s \vec{D} \cdot \vec{ds} = \int_v \rho_v \cdot dv$.
iv) $\nabla \cdot \vec{B} = 0$.	iv) $\oint_s \vec{B} \cdot \vec{ds} = 0$.

(2) In a given lossy dielectric medium, conduction current density $J_c = 0.02 \sin 10^9 t$ (A/m²). Find the displacement current density if $\sigma = 10^3$ S/m & $\epsilon_r = 6.5$. (11)

:- $$\left|\frac{J_c}{J_D}\right| = \frac{\sigma}{\omega\epsilon}$$

$$J_D = \frac{\omega\epsilon J_c}{\sigma} = \frac{10^9 \times 8.85 \times 10^{-12} \times 6.5 \times 0.02}{10^3}$$

$$= 1.151 \times 10^{-6}\ \text{A/m}^2 = \underline{1.151 \mu\ \text{A/m}^2}.$$

As $\vec{J_D}$ & $\vec{J_c}$ are always at right angles to each other we can write.

$$\vec{J_D} = \underline{1.151 \times 10^{-6}} \cos 10^9 t\ \ \text{A/m}^2.$$

(3) If the magnetic field $\vec{H} = [3x\cos\beta + 6y\sin\alpha]\hat{a}_z$, find current density $\vec{J}$ if fields are invariant with time.

:- the point form of maxwell's second equation is $$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}.$$

Since field is time invariant.

$$\frac{\partial \vec{D}}{\partial t} = 0.$$

$$\nabla \times \vec{H} = \vec{J}.$$

$$\vec{J} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 3x\cos\beta + 6y\sin\alpha \end{vmatrix}$$

$$\vec{J} = \hat{a}_x\left[\frac{\partial}{\partial y}(3x\cos\beta + 6y\sin\alpha)\right] - \hat{a}_y\left[\frac{\partial}{\partial x}(3x\cos\beta + 6y\sin\alpha)\right]$$

$$\vec{J} = 6\sin\alpha\,\hat{a}_x - 3\cos\beta\,\hat{a}_y \quad \text{A/m}^2.$$

(4) A circular loop of 10 cm radius is located in the x-y plane in a $\vec{B}$ field is given by $\vec{B} = 0.5\cos(377t)(3\hat{a}_y - 4\hat{a}_z)$ T. Determine the voltage induced in the loop.

:- $$e = -\frac{d\phi}{dt}.$$

$$\phi = \int_s \vec{B}\cdot\vec{ds} = \int_{r=0}^{0.1}\int_{\phi=0}^{2\pi} 0.5\cos(377t)(3\hat{a}_y - 4\hat{a}_z)\cdot(r\,dr\,d\phi\,\hat{a}_z)$$

$$= 0.5\cos(377t)(-4)\int_{r=0}^{0.1} r\,dr \int_{\phi=0}^{2\pi} d\phi.$$

$$= 0.5\cos(377t)(-4)\left[\frac{r^2}{2}\right]_0^{0.1}[2\pi].$$

$$= -4\times 0.5\cos(377t)\left[\frac{0.1^2}{2}\right](2\pi)$$

$$\phi = 0.0628\cos(377t).$$

$$e = -\frac{d\phi}{dt} = -0.0628\times 377\sin(377t).$$

$$\boxed{e = 23.69\sin(377t)}$$

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(5) Calculate the voltage across 1Ω & 2Ω resistor shown in fig. The loop is located in the XY-plane & $\phi = 0.1t$ wb.

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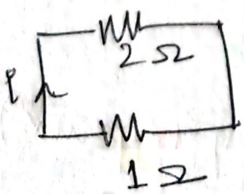


$$I = \frac{\text{induced emf}}{\text{resistor}}.$$

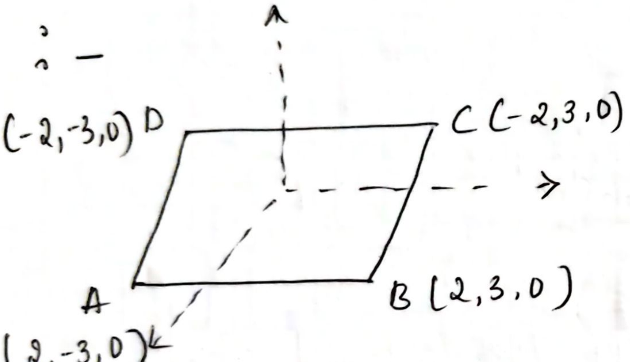
$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(0.1t) = -0.1$$

$$|e| = 0.1.$$

$\Rightarrow i = \underline{0.01}$

(6) Given $\vec{B} = (0.5\hat{a}_x + 0.6\hat{a}_y - 0.3\hat{a}_z)\cos(5000t)$ T & a filamentary loop with its corners at $(2,3,0)$ m, $(2,-3,0)$ m, $(-2,-3,0)$ & $(-2,3,0)$ m. Find the emf developed in the loop.

:-



$$e = -\frac{d\phi}{dt}.$$

$$\phi = \int_s \vec{B}\cdot\vec{ds}$$

length AB = 6m, length AD = 4m.

Surface area = 6x4 = 24.

& is ⊥r to z axis

$$\therefore \phi = \int_s \vec{B}\cdot\vec{ds} = \vec{B}\cdot\vec{S}.$$

$$= \cos 5000t\,(0.5\hat{a}_x + 0.6\hat{a}_y - 0.3\hat{a}_z)(24\hat{a}_z).$$

$$\phi = -0.3\times 24\cos 5000t := 7.2\cos 5000t.$$

$$e = -\frac{d\phi}{dt} = \frac{d}{dt}(7.2\cos 5000t) = 7.2\times 5000\sin 5000t.$$

$$e. = -36\times 10^3 \sin 5000t.$$

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(7) An area of $0.65\,m^2$ in the plane $z=0$ encloses a filamentary conductor. Find the induced voltage if $\vec{B} = 0.05\cos 10^3 t\left[\frac{\hat{a}_y + \hat{a}_z}{\sqrt{2}}\right]$ T.

:- $e = -\dfrac{d\phi}{dt}.$

$$= -\frac{d}{dt}\int_s \vec{B}\cdot\vec{ds}$$

$$= -\int_s \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}$$

$$= -\int_s \frac{\partial}{\partial t}\left[0.05\cos 10^3 t\left(\frac{\hat{a}_y+\hat{a}_z}{\sqrt{2}}\right)\right]\cdot(ds\,\hat{a}_z)$$

$$= -\int_s \frac{\partial}{\partial t}\left[\frac{0.05\cos 10^3 t}{\sqrt{2}}\right]\cdot ds$$

$$= -\int_s \frac{0.05\times 10^3(-\sin 10^3 t)}{\sqrt{2}}\cdot ds$$

$$= 35.355\sin 10^3 t\cdot\left[\int_s ds\right]$$

But $\int_s ds$ is given as $0.65\,m^2$.

$$= 35.355\sin 10^3 t\,(0.65)$$

$$e = 22.98\sin 10^3 t \text{ V}.$$

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(8). In a material for which $\sigma = 5.0$ s/m & $\epsilon_r = 1$, the electric field intensity is $E = 250\sin 10^{10} t$ V/m. Find the conduction & displacement current density & the frequency at which both have equal magnitudes.

:- $J_c = \sigma E$

$$= 5(250\sin 10^{10} t) = 1250\sin 10^{10} t \text{ A/m}^2.$$

$$J_D = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\epsilon E) = \frac{\partial}{\partial t}(\epsilon_0\epsilon_r E).$$

$$= \frac{\partial}{\partial t}(8.854\times 10^{-12}\times 1\times 250\sin 10^{10} t)$$

$$= (8.854\times 10^{-12}\times 250)(10^{10})\cos(10^{10} t).$$

$$J_D = 22.135\cos 10^{10} t\ \text{A/m}^2.$$

For the two densities, the condition for magnitudes to be equal is . (15)

$$\frac{|\vec{J_c}|}{|\vec{J_D}|} = \frac{\sigma}{\omega\epsilon} = 1.$$

$$\omega = \frac{\sigma}{\epsilon} = \frac{5}{8.854\times10^{-12}\times1} = \underline{\underline{5.6471\times10^{11}}}.$$

$\omega = 2\pi f$.

$$f = \frac{\omega}{2\pi} = \frac{5.647\times10^{11}}{2\pi} = \underline{\underline{89.87\,GHz}}.$$

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9) a) Show that the ratio of the amplitudes of the conduction current density & displacement current density is $\frac{\sigma}{\omega\epsilon}$, for the applied field $E = E_m\cos\omega t$. Assume $\mu = \mu_0$.

b) What is the amplitude ratio of the applied field is $E = E_m e^{-t/\tau}$ where τ is real.

:- a) $J_c = \sigma E = \sigma E_m\cos\omega t$.

$$J_D = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\epsilon E) = \epsilon\frac{\partial}{\partial t}(E_m\cos\omega t).$$

$$J_D = -\omega\epsilon E_m\cos\omega t.$$

The ratio of the magnitudes

$$\frac{|\vec{J_c}|}{|\vec{J_D}|} = \frac{\sigma E_m\cos\omega t}{-\omega\epsilon E_m\cos\omega t} = \frac{\sigma}{\omega\epsilon}.$$

b) $E = E_m e^{-t/\tau}$.

$$J_c = \sigma E = \sigma E_m e^{-t/\tau}.$$

$$J_D = \epsilon\frac{\partial}{\partial t}\left[E_m e^{-t/\tau}\right] = \epsilon E_m\left(\frac{-1}{\tau}\right)e^{-t/\tau} = \frac{-\epsilon E_m}{\tau}e^{-t/\tau}$$

$$\frac{|\vec{J_c}|}{|\vec{J_D}|} = \frac{\sigma E_m e^{-t/\tau}}{\frac{\epsilon E_m}{\tau}e^{-t/\tau}} = \underline{\underline{\frac{\sigma\tau}{\epsilon}}}.$$

10) Find the amplitude of the displacement current density a) $\vec{E} = 80\cos(6.277\times10^8 t - 2.092y)\hat{a}_{y}$ V/m, $\epsilon_r = 1$.

b) $\epsilon_r = 600$ & $\vec{D} = 3\times10^{-6}\sin(6\times10^8 t - 0.3464x)\hat{a}_z$ C/m^2.

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:- a) $\vec{J}_D = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\epsilon_0 \epsilon_r E)$.

$= \epsilon_0 \frac{\partial}{\partial t}[80\cos(6.277\times10^8 t - 2.092y)\hat{a}_z]$.

$= 8.854\times10^{-12}\times 80\times(-6.277\times10^8)\sin(6.277\times10^8 t - 2.092y)\hat{a}_z$

$\vec{J}_D = -0.4446\sin(6.277\times10^8 t - 2.092y)\hat{a}_z$ A/m^2.

amplitude $= -0.4446$ A/m^2.

b) $\vec{J}_D = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}[$

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11) Given $\mu = 10^{-5}$ H/m, $\epsilon = 4\times10^{-9}$ F/m, $\sigma = 0$, $\rho_v = 0$. Find K so that following pairs of fields satisfy maxwell's equations

i) $\vec{D} = 6\hat{a}_x - 2y\hat{a}_y + \cancel{2\hat{a}_z}\, 2z\hat{a}_z$

$\vec{H} = Kx\hat{a}_x + 10y\hat{a}_y - 25z\hat{a}_z$.

ii) $\vec{E} = (20y - kt)\hat{a}_x$

$\vec{H} = (y + 2\times10^6 t)\hat{a}_z$.

:- i) $\nabla\cdot\vec{D} = \rho_v$,

As per data $\rho_v = 0$.

$\nabla\cdot\vec{D} = 0$.

$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0$. —(1)

$D_x = 6$, $D_y = -2y$, $D_z = 2z$ } substitute in eq (1)

$0 - 2 + 2 = 0 = \nabla\cdot\vec{D}$.

It satisfies maxwell's equation.

$\nabla\cdot\vec{B} = 0$

$\nabla\cdot\mu\vec{H} = 0$

$\mu(\nabla\cdot\vec{H}) = 0$.

Since $\mu \neq 0$.

$\nabla\cdot\vec{H} = 0$

$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$, -(2)

$H_x = Kx$, $H_y = 10y$, $H_z = -25z$ } substitute in eq(2)

$K + 10 - 25 = 0$.

$\boxed{K = 15}$